

1. Solve the 2nd order differential equation (15%)

$$y'' - 6y' + 9y = 0, \quad y(0) = 0, \quad y'(0) = 1.$$

2. Given the 2nd order inhomogeneous differential equation (15%)

$$y''(x) - 6y'(x) + 9y(x) = e^{3x},$$

use the method of variation of parameters to find the particular solution.

3. Solve the 1st order linear differential equation (10%)

$$y' + y = xe^{-x}.$$

4. If $\mathbf{A} = \begin{bmatrix} 5 & -6 & 7 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 3 \\ 4 \\ -1 \end{bmatrix}$, and $\mathbf{C} = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & -1 \\ 3 & 2 & 1 \end{bmatrix}$, find (a) \mathbf{BA} , (b) $(\mathbf{BA})\mathbf{C}$. (20%)

5. Find the eigenvalues and eigenvectors of (20%)

$$\mathbf{A} = \begin{bmatrix} 5 & 6 \\ 3 & -2 \end{bmatrix}.$$

6. Evaluate $\int_C xydx + x^2dy$, where C is given by $y = x^3$, $-1 \leq x \leq 2$. (20%)